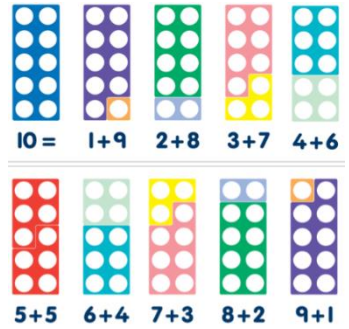
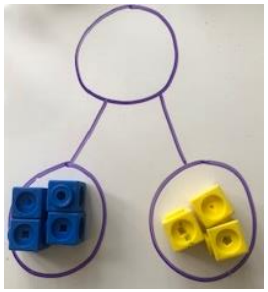
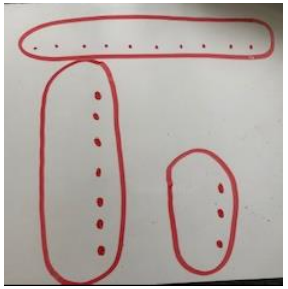
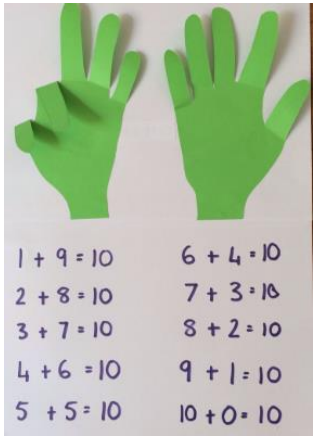
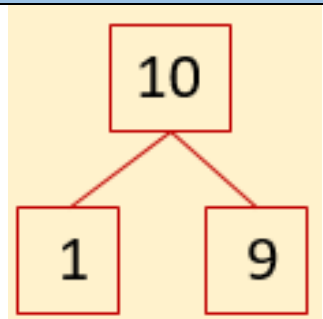
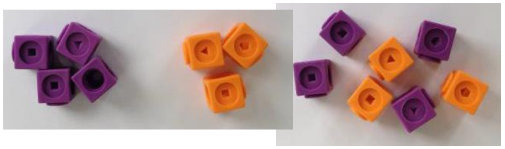
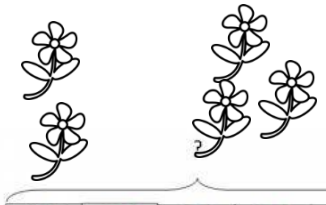
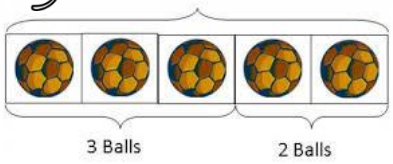
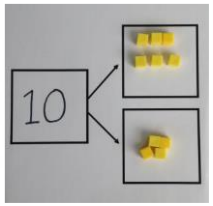
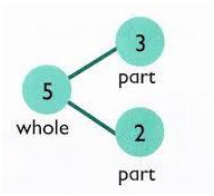
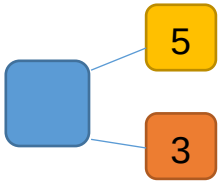
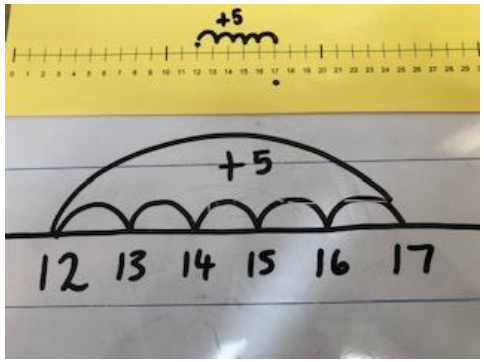
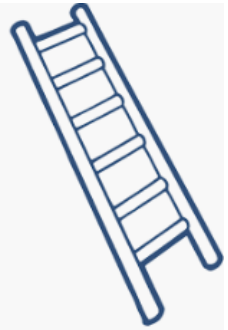
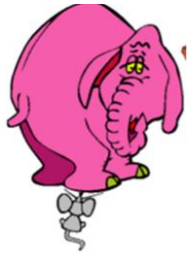
Mathematics Calculation Policy

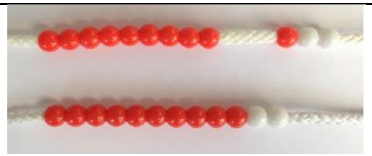
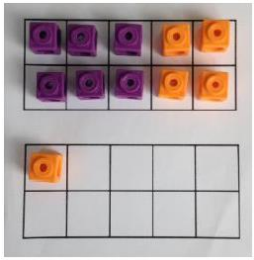
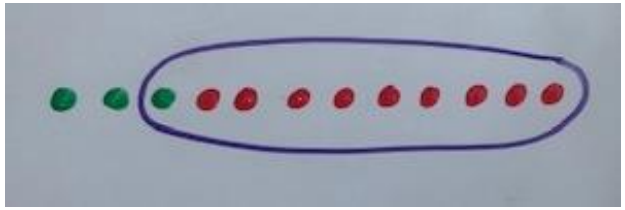
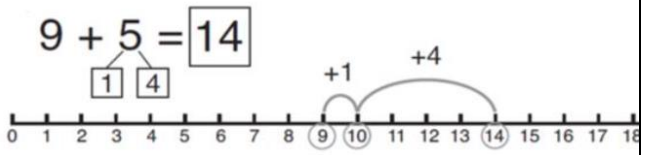
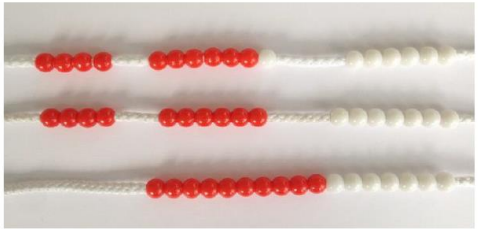
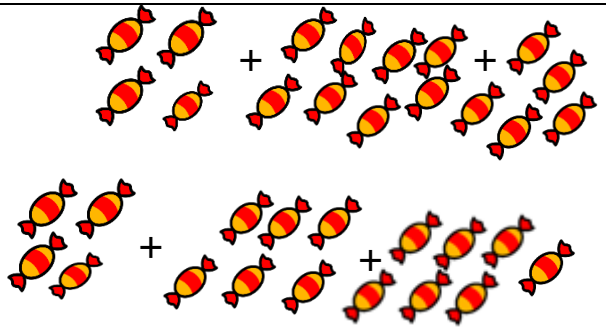
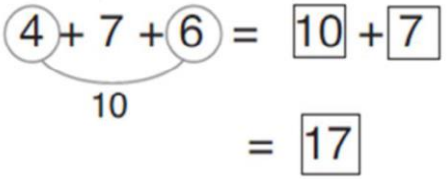
This policy is a statement of the aims, principles and strategies for teaching and learning of calculation strategies in Mathematics. At Elstow School we firmly believe in a mastery approach and use resources published by White Rose and NECTEM who are leaders in the field of Mastery in Mathematics. This policy ensures that calculation is taught consistently across the school and that all staff are familiar with the most precise and effective methods including use of resources and manipulatives. The methods build understanding and confidence through learning in small steps. This policy is also designed to help parents and carers support children's learning by providing an explanation of the methods used, in order for them to feel confident to assist in their children's learning. The policy shows the progression of skills, knowledge and layout for written methods, set out in the following strands: addition & subtraction; multiplication & division.

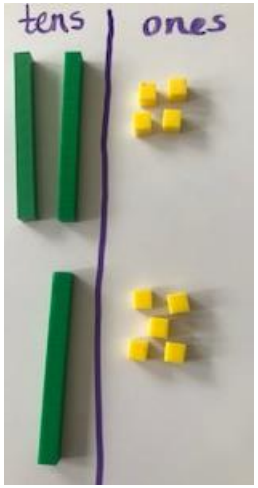
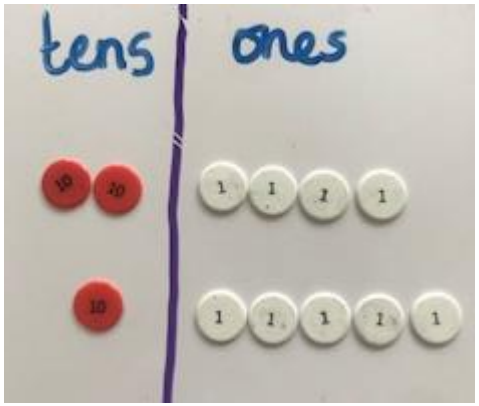
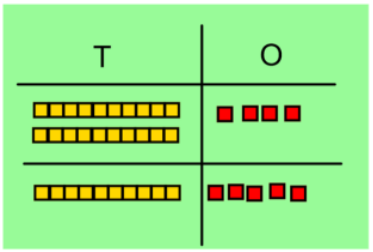
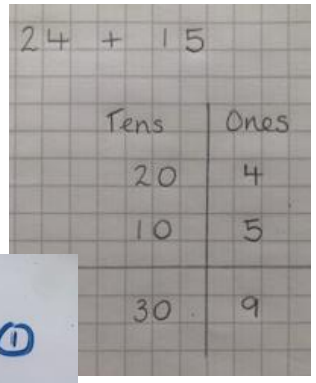
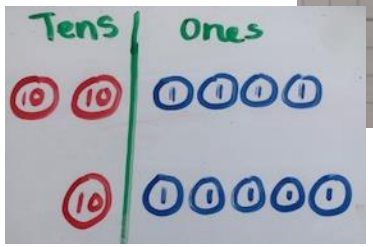
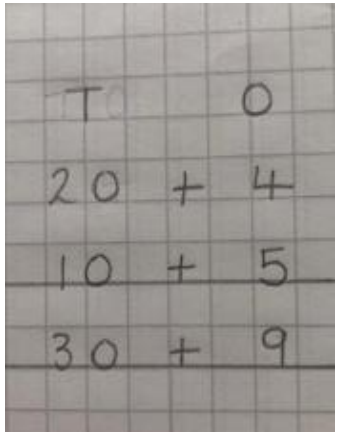
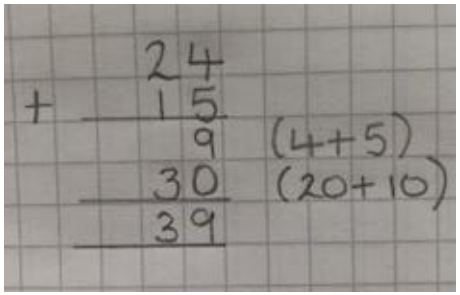
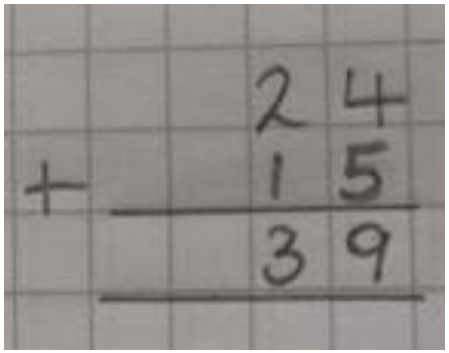
We follow a Concrete – Pictorial - Abstract model (CPA) which starts with a mathematical concept demonstrated through manipulatives (cubes, beads, counters) and then a pictorial representations (such as bar models or arrays) which is used simultaneously with the concrete. This ensures a full understanding of the concept, before linking it to the abstract form (operation signs, equations). We recognise the importance of the use of correct and precise mathematical language and believe that all children are capable of learning mathematical terms. Mental calculation methods are complementary to written methods and should not be seen as separate from them. Children are taught how to mentally calculate efficiently and with accuracy. A demonstration of mastery is a reflection of child's ability to know whether a calculation can be done mentally or requires a written method. The steps in the policy below are intended to support understanding of how to calculate and how these are taught at Elstow.

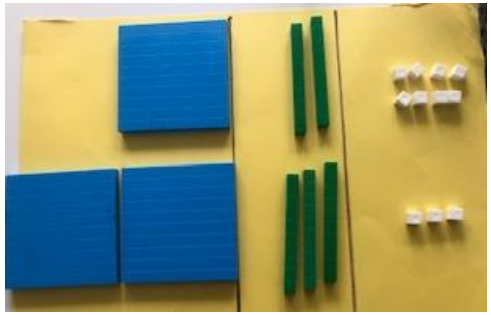
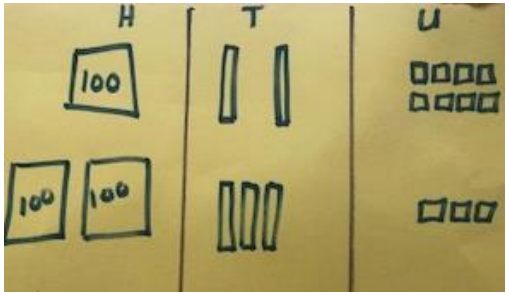
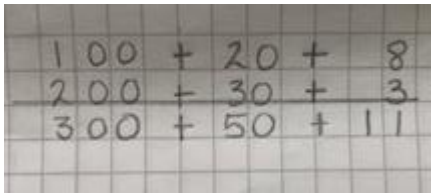
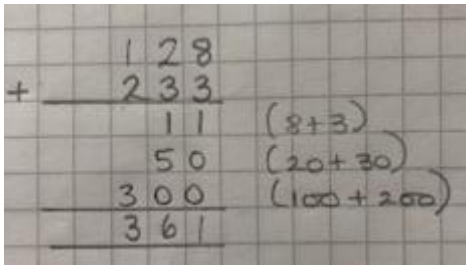
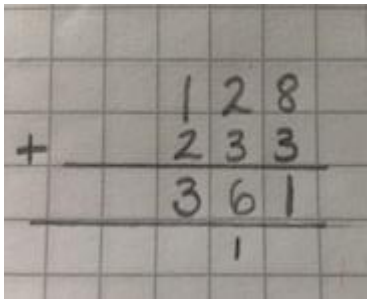
Addition

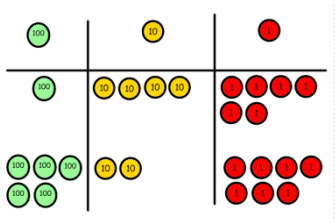
Objective and Strategies	Concrete	Pictorial	Abstract including mental strategy.
Recall and use addition and subtraction facts for all numbers up to and within 10. Having a secure recall of compliments to 10 or number bonds is an essential skill forming the basis for further calculation.	   Introduce idea of 2 parts combining to make a whole amount.	Drawing a dot for every cube, then removing the cubes.   Reinforce idea of 2 parts combining to make a whole amount using a bar model. **10** **1** | **9**	 1 + 9 = 10 9 + 1 = 10 10 – 1 = 9 10 – 9 = 1 Learn the compliments in sets of 4 so that the inverse facts become embedded learning.

Combining two parts to make a whole: part- whole model	  Use cubes to add two numbers together as a group or in a bar.	 Use pictures to add two numbers together as a group or in a bar.      	Use the part-part whole diagram to move into the abstract. 4 + 3 = 7 10 = 6 + 4
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number one by one to find the answer.	12 + 5 = 17  	5 + 12 = 17 Elephant and mouse rule: Place the larger number in your head and count on the smaller number to find your answer.  

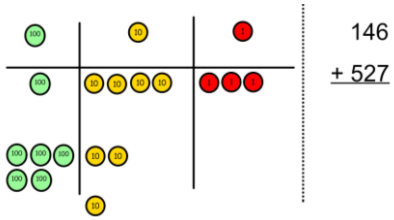
Regrouping to make 10.	 6 + 5 = 11  Start with the larger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10? How many more do I add on now?
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remaining number.

Column method- **no** regrouping	24 + 15= Add together the ones first then add the tens. Use the **Base 10 blocks** first before moving onto **place value counters**.  	Children can pictorially represent the counters to represent the calculation.    	Expanded method adds the ones, then the tens.  Leading to succinct column layout. 

Column method- **regrouping**	Make both numbers on a place value grid Introduce using Base 10, then move onto place value counters when the children are secure. 	Children can draw a pictoral representation of the columns and place value counters. Further support the understanding of plave value by using the arrow cards beneath the pictures.   Then recombine to form the numbers. 	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. Start with a direct representation of the place value grid.   



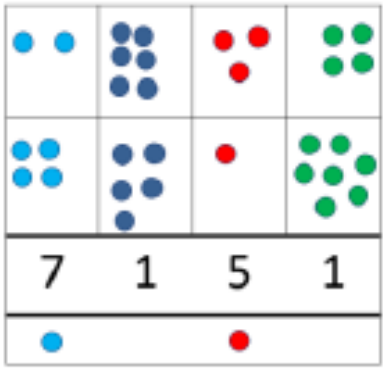
Add up the units and exchange 10 ones for one 10.



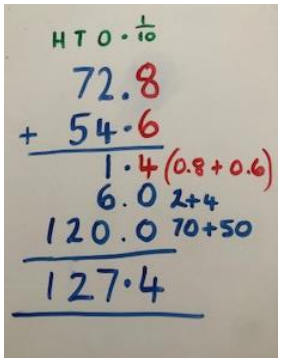
Add up the rest of the columns, **exchanging** the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

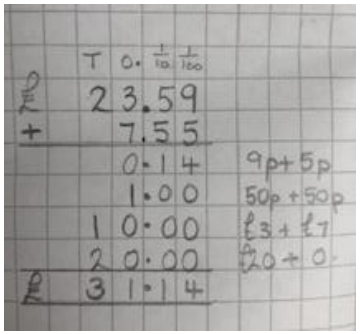


As the children move on, introduce decimals with the same number of decimal places and different, including money and measures.

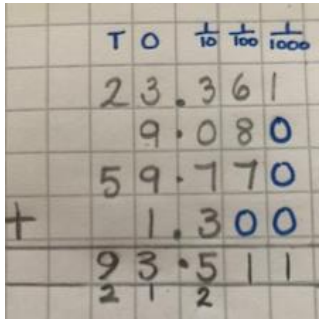


Expanded method with tenths.

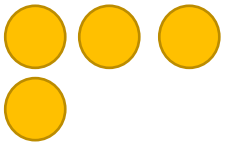
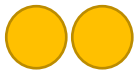
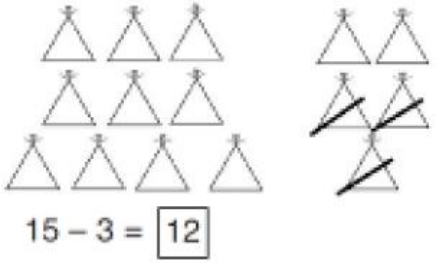
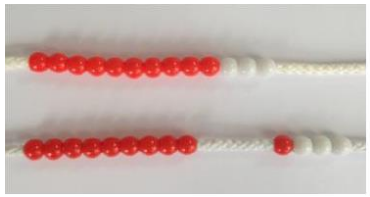
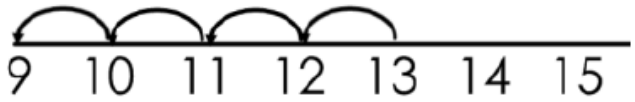
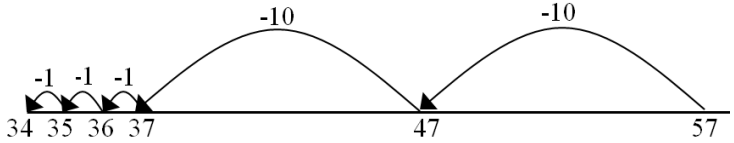
Expanded method with tenths and hundredths, such as money

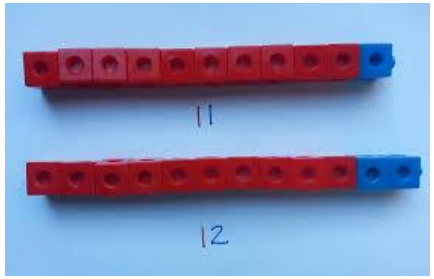
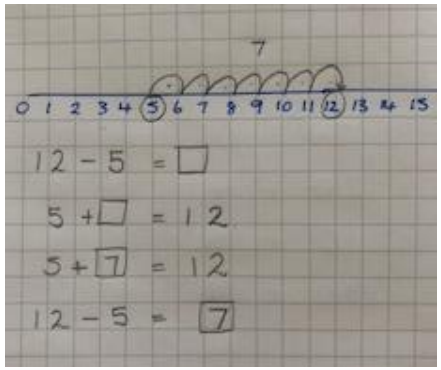
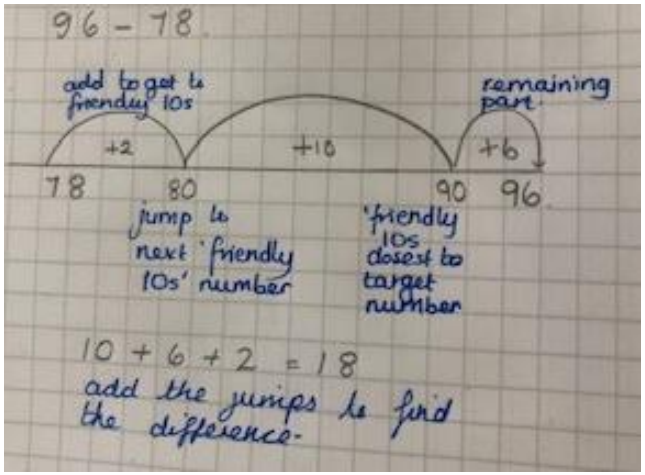


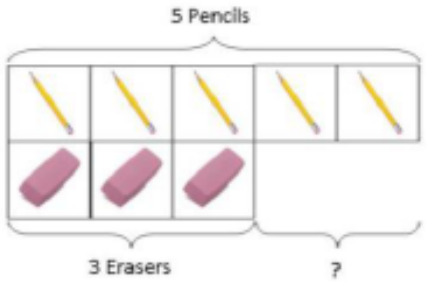
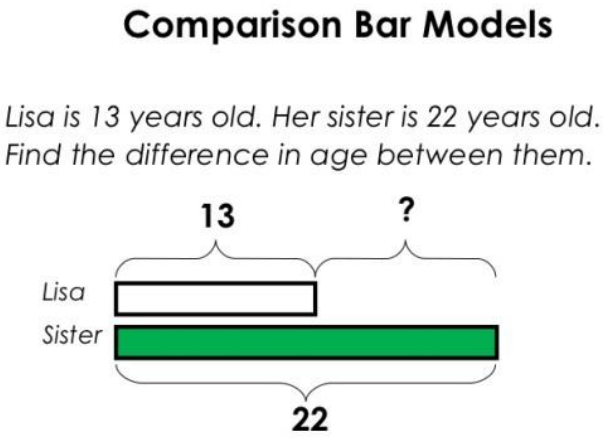
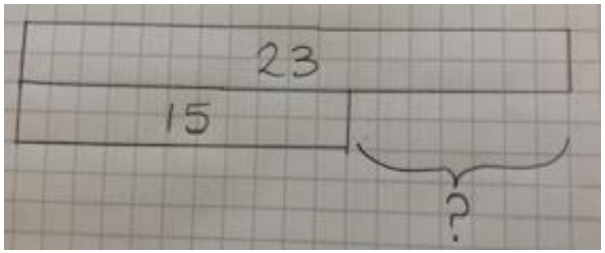
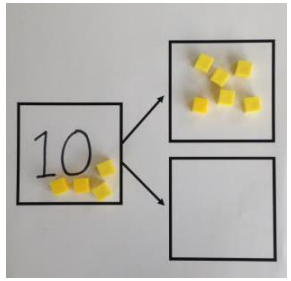
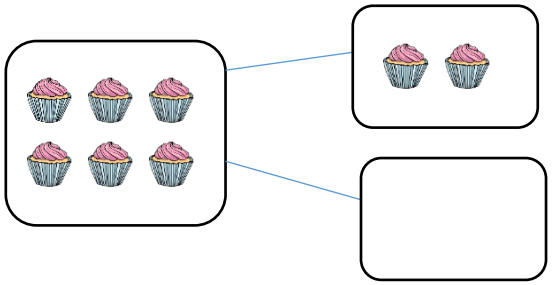
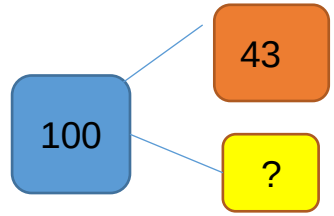
When adding values with differing decimal places, pupils will add the place holding zeros to support the correct use of columns in succint additon methods.

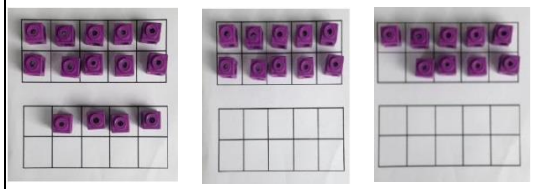
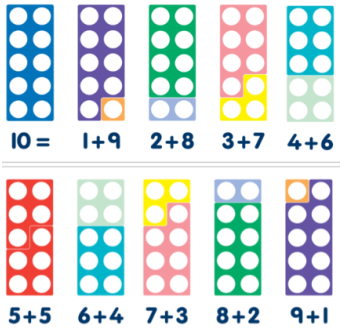
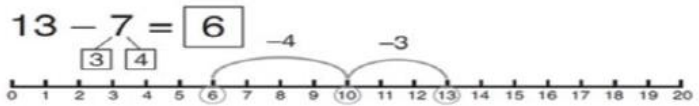
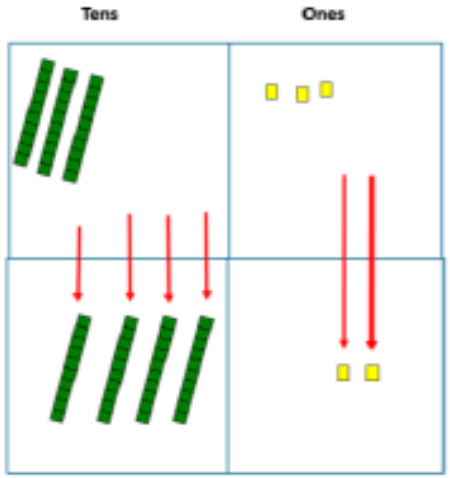
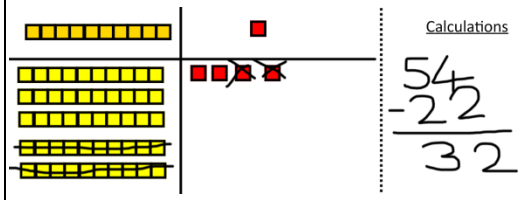
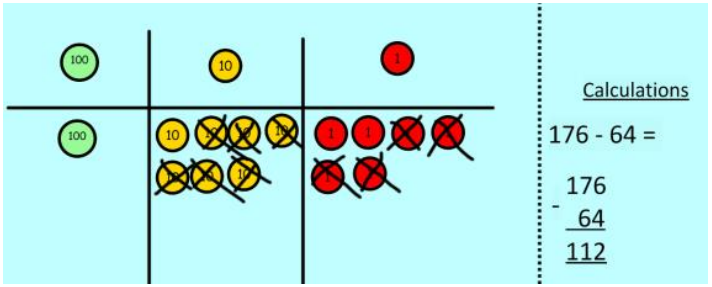
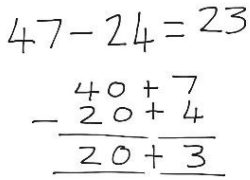
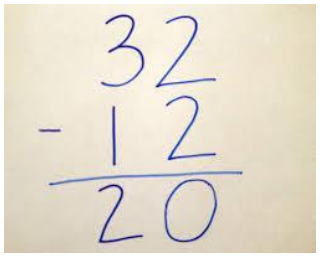


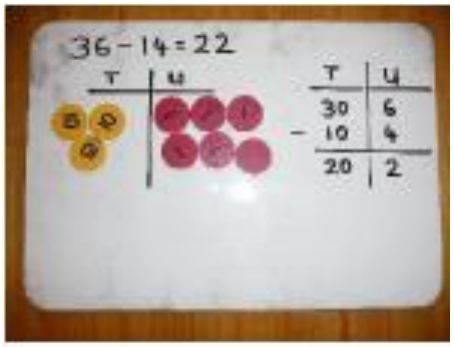
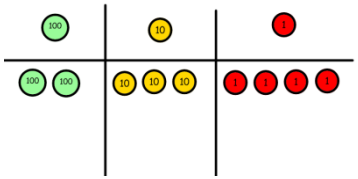
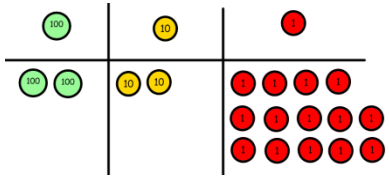
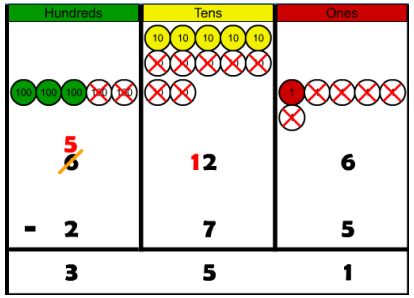
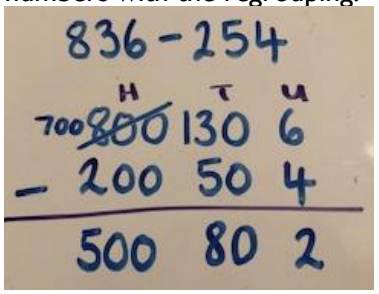
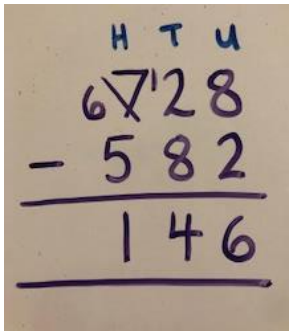
Subtraction

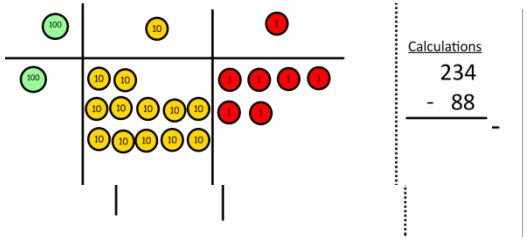
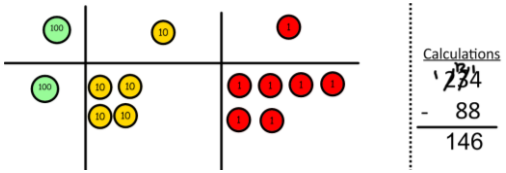
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, such as counters and cubes to show how objects can be taken away.  6 – 2 = 4  	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	8 – 2 = 6 18 – 3= 15 20 = 23 – 3 17 – ☐ = 15
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers, by partitioning into tends and ones and counting back each ten, then the ones.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. Don't count the number you are starting on as your first number.

Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Which leads to **COMPARISON BAR MODELS**	Use number lines to count on to find the difference.   	Hannah has made 23 sandwiches, Helen made 15 sandwiches. How many more sandwiches did Hannah make?

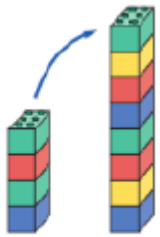
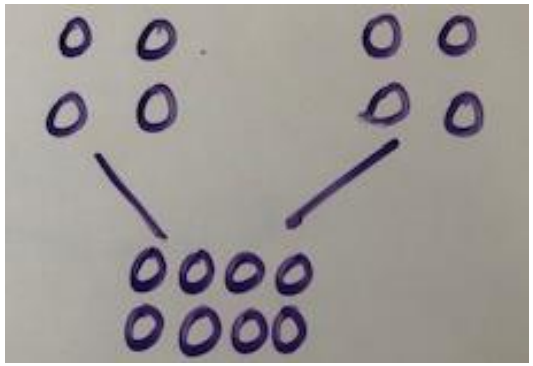
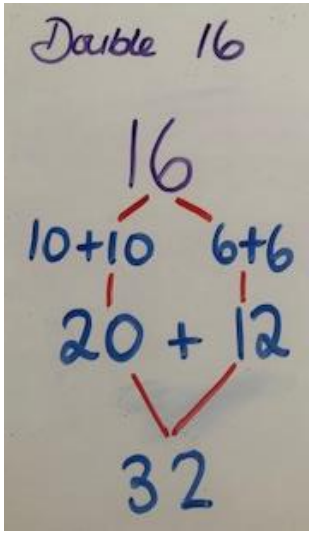
	Use basic bar models with items to find the difference  	 Draw bars to find the difference between 2 numbers.  	
Part- Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part-whole model. 	 Move to using numbers within the part whole model alongside the abstract calculation. 100 – 43 = ?

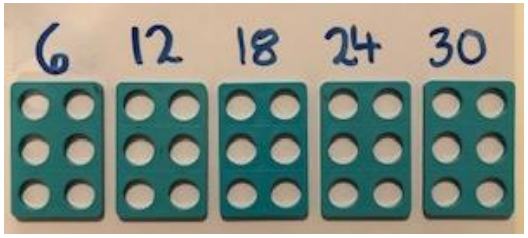
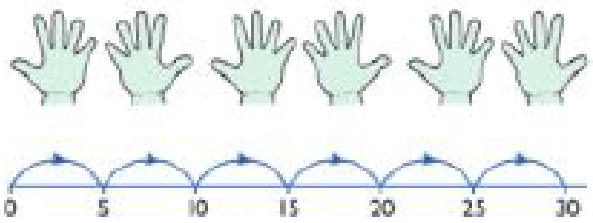
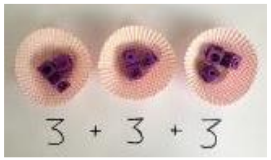
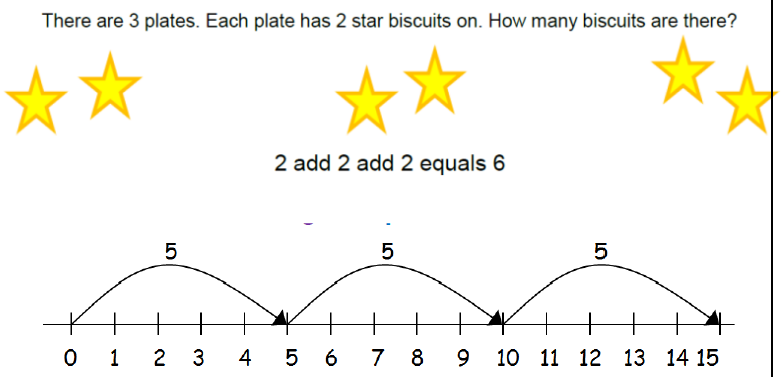
Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9. 	Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. 	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Column method **without regrouping**	 Use Base 10 to make the bigger number then take the smaller number away.	 Draw the Base 10 or place value counters along side the written calculation to help to show working. 	 This will lead to a secure understadning of place value in written column subtraction. 

	 Show how you partition numbers to subtract. Again **make the larger number first.**		
Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Calculations 234 - 88 Start with the ones: can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Calculations 234 - 88 Now I can subtract my ones.	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup. Moving forward the children use a more compact method.	Children can start their formal written method by the number into clear place value columns. Show the partitioned numbers with the regrouping.  Three digits with regrouping a hunded into 10 tens. 

	Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling Support understanding of 'the same again'.	Use practical activities to show what it means to double a number.  double 4 is 8 4 × 2 = 8 Use a mirror.  Use symmetry 	Draw pictures to show how to double a number. 	Partition a number and then double each part before recombining back together. 

Counting in multiples	   Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30 Which of these numbers is not a multiple of 4? 40, 48, 56, 65 84
Repeated addition It is fine to introduce the concept of 'multiples of a number' as repeated addition of that number, but this is only one way of looking at multiplication and only applies to positive whole numbers.	Use different objects to add equal groups.  	 	Write addition sentences to describe objects and pictures. 

Arrays- showing commutative multiplication.

It is important use the correct terminology in multiplication, as part of rich mathematical knowledge.

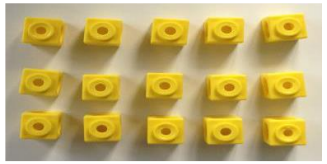
Multiplicand- the number in our set.

Multiplier – the number of sets we have.

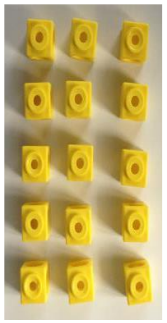
They multiply together to make a **product**.

Both multiplier and multiplicand are known as **factors** of that product.

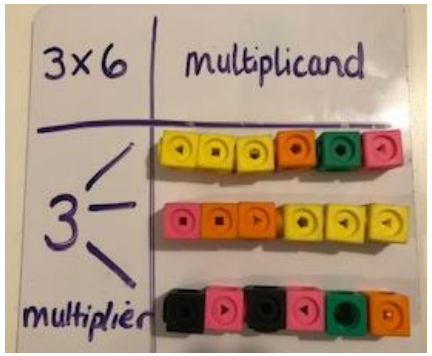
Create arrays using counters, cubes and other to show multiplication sentences.



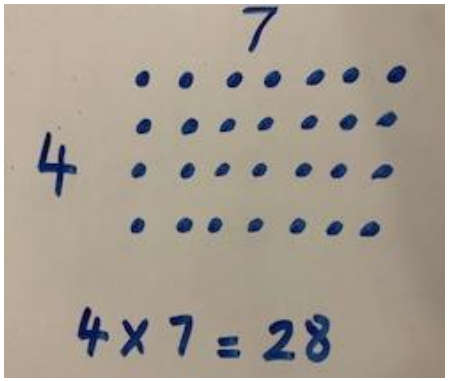
This array shows 3 lots of 5.



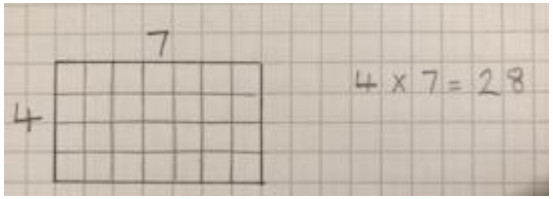
It is displayed here as 5 lots of 3



Draw arrays in different rotations to find **commutative** multiplication sentences.



Link arrays to area of rectangles.



Pupils should have plenty of experience of arrays to embedd understanding of how multiplication increases natural numbers rapidly,

Use an array to write multiplication sentences and reinforce repeated addition.

5 + 5 + 5 = 15

3 + 3 + 3 + 3 + 3 = 15

3 x 5 = 15

5 x 3 = 15

Check understanding beyond learned facts with questioning.

What are the factors of 15?

Can 8 be a factor of 18? How do you know?

Finding factors

Being able to **factorize** a **composite numbe**r into its component factors is an essential maths skill and leads into understanding of **prime numbers**.

How many different ways can you rearrange 12 cubes to form a rectangle?

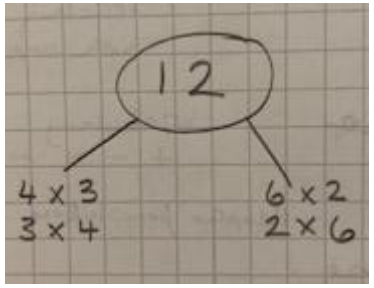
3 X 4 4 X 3

2 X 6

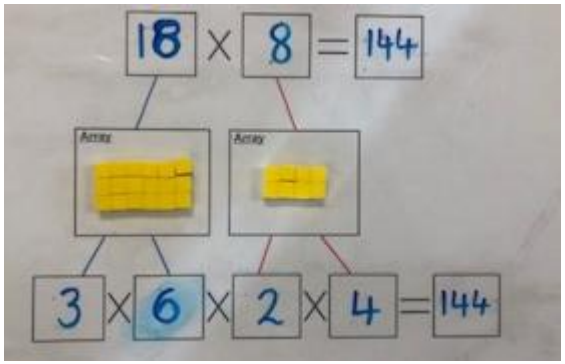
6 X 2

Draw the different arrays to show the factors.

List the factors in pairs.



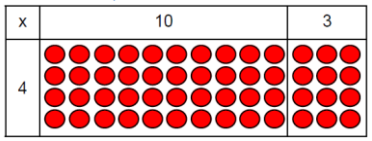
Once pupils are confident to find factors this can be applied to larger composite numbers



Grid Method

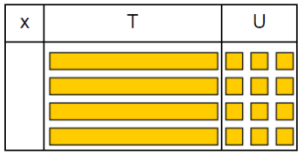
Misconceptions with place value can be avoided by using Base 10 and place value counters before numbers.

Show the link with arrays to first introduce the grid method.



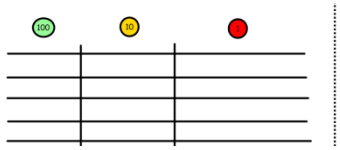
4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

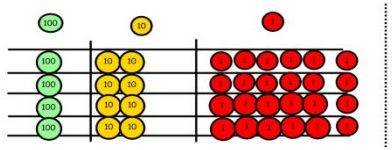


4 rows of 13

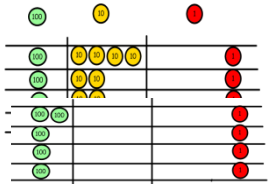
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



Fill each row with 126.



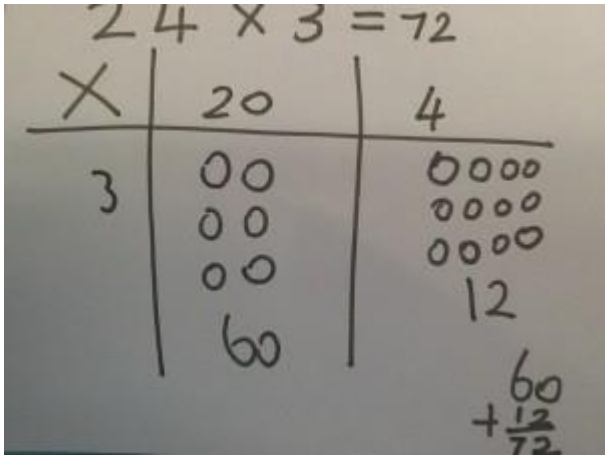
Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

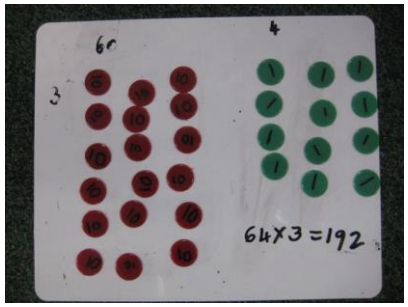
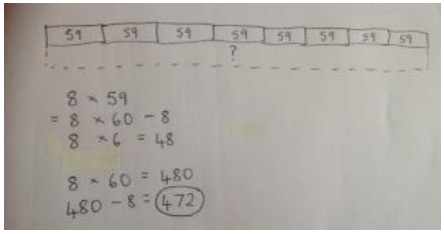
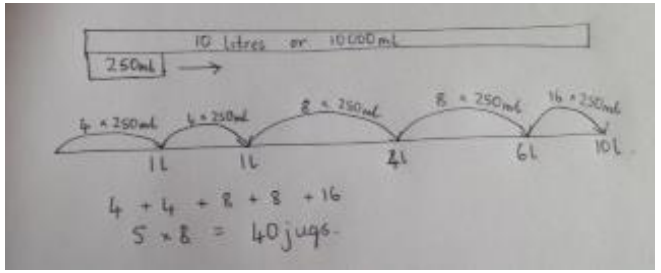
×	30	5
7	210	35

210 + 35 = 245

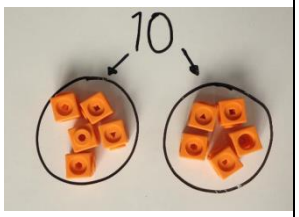
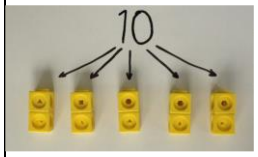
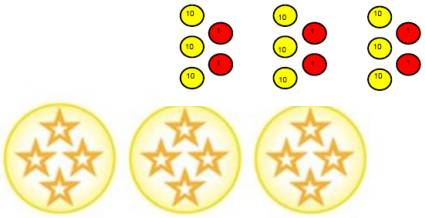
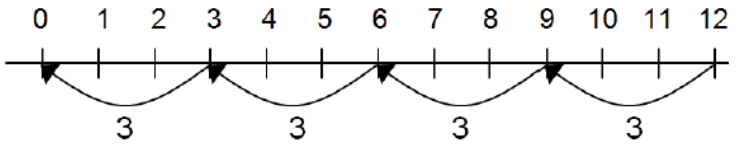
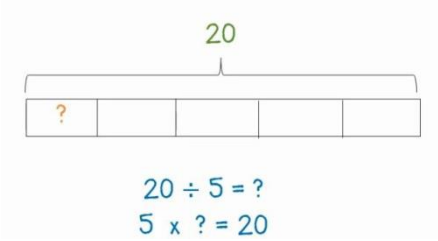
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.    	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 74 x 63 12 210 240 + 4200 4662 2 3 1 1342 x 18 13420 10736 24156 1 This moves to the more compact method.

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups?  	Children use pictures or shapes to share quantities.   8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

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Division within arrays

Dividend= the number to be divided.

Divisor = the number it is divided by.

Quotient = the answer.

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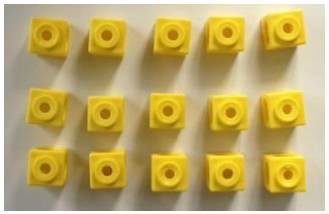

Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg 15 ÷ 3 = 5 5 x 3 = 15
15 ÷ 5 = 3 3 x 5 = 15

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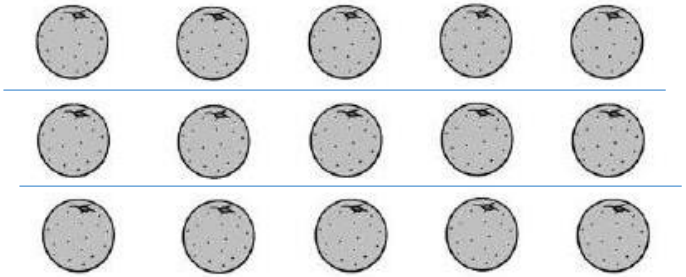

Draw an array and use lines to split the array into groups to make multiplication and division sentences.

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Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7

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Division with a remainder

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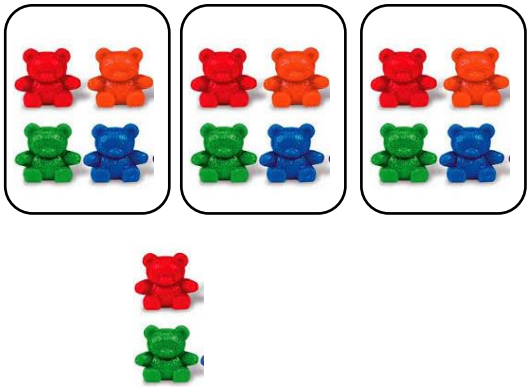
14 ÷ 3 =
Divide objects between groups and see how much is left over



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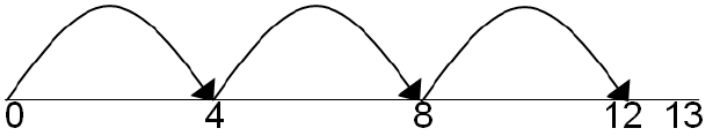
Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.

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Complete written divisions and show the remainder using r.

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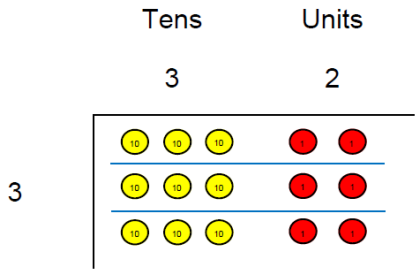
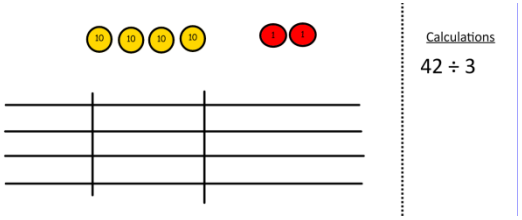
Short division

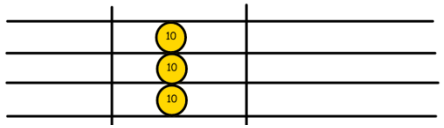


Use place value counters to divide using the bus stop method alongside

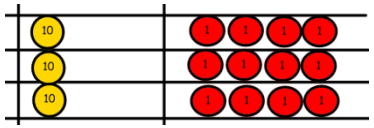


42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

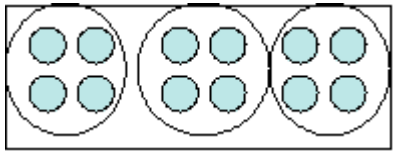


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3\overline{)872} \end{array}$$

Move onto divisions with a remainder.

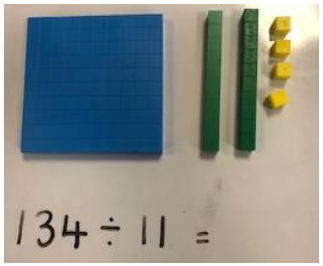
$$\begin{array}{r} 86\ r\ 2 \\ 3\overline{)432} \end{array}$$

Finally move into decimal places to divide the total accurately.

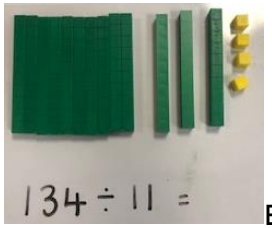
$$\begin{array}{r} 14.6 \\ 35\overline{)511.0} \end{array}$$

Long division

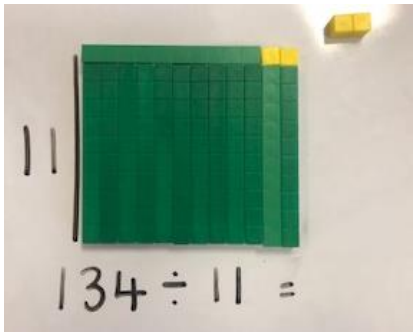
Use Base 10 to make the dividend.



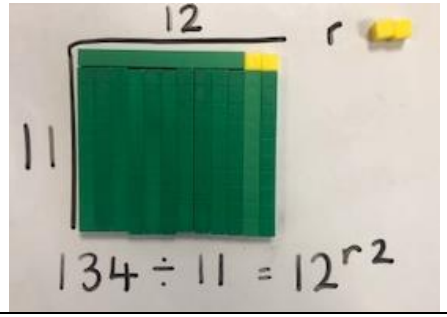
Can you make a perfect ractangle with the Base 10?



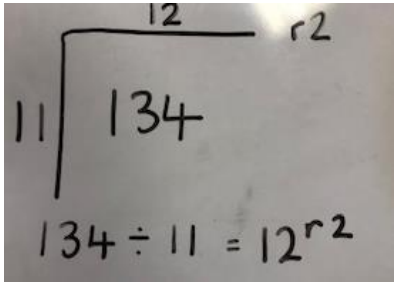
Exchange larger denominations for smaller ones.

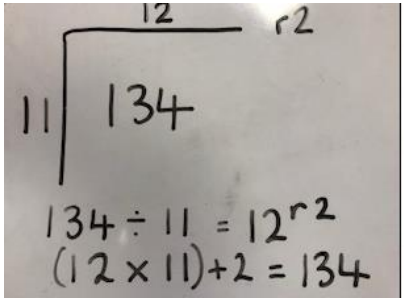


Lay the base 10 out in a rectangle with the **divisor as the width.** Draw in a line next to the divisor.



When you remove the Base 10, you can clearly see the bus stop layout.

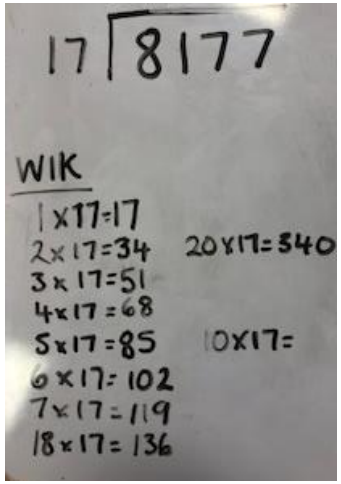




Pupils then check using the inverse.

Always engourage pupils to write a WIK box (What I know) in multiples of the divisor to help before long division.

This helps them to estimate an answer and spot errors more easily.



	Count how many you have as the length of the rectangle. Are there any left over that cannot form a a triangle? This is your **quotient** and **remainder.**